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A Method for Estimating Depth-Limited Wave Energy

by
C. Linwood Vincent

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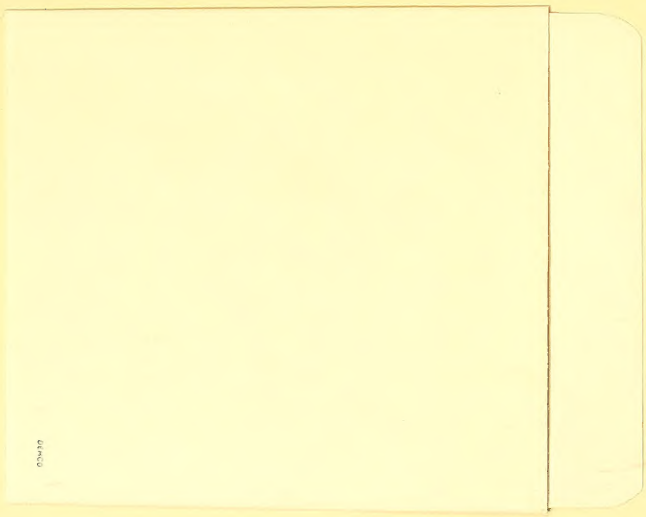

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A method for estimating an upper limit of wind wave energy in shallow water is presented. The method requires knowledge of the depth, the peak frequency of the sea, and the windspeed in order to predict a depth-controlled wave height, H, defined as $4(E)^{1/2}$, with E the energy of the wind sea. In the shallow limit, H is shown to be approximately proportional to the square root of depth. The method is recommended for predictions in storm seas and not for swell (i.e., nearly monochromatic waves).

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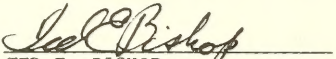
PREFACE

This report presents a method for estimating an upper limit on wave energy in shallow water as a function of depth and parameters of the wave spectrum. The research was carried out under the coastal waves and flooding program of the U.S. Army Coastal Engineering Research Center (CERC).

The report was prepared by Dr. C. Linwood Vincent, Chief, Coastal Oceanography Branch, under the general supervision of R.P. Savage, Chief, Research Division. J.E. McTamany prepared the computer integration scheme; W.N. Seelig and L.L. Broderick provided the laboratory data.

Comments on this publication are invited.

Approved for publication in accordance with Public Law 166, 79th Congress, approved 31 July 1945, as supplemented by Public Law 172, 88th Congress, approved 7 November 1963.



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CONVERSION FACTORS, U.S. CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT

U.S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

Multiply	by	To obtain
inches	25.4	millimeters
	2.54	centimeters
square inches	6.452	square centimeters
cubic inches	16.39	cubic centimeters
feet	30.48	centimeters
	0.3048	meters
square feet	0.0929	square meters
cubic feet	0.0283	cubic meters
yards	0.9144	meters
square yards	0.836	square meters
cubic yards	0.7646	cubic meters
miles	1.6093	kilometers
square miles	259.0	hectares
knots	1.852	kilometers per hour
acres	0.4047	hectares
foot-pounds	1.3558	newton meters
millibars	1.0197×10^{-3}	kilograms per square centimeter
ounces	28.35	grams
pounds	453.6	grams
	0.4536	kilograms
ton, long	1.0160	metric tons
ton, short	0.9072	metric tons
degrees (angle)	0.01745	radians
Fahrenheit degrees	5/9	Celsius degrees or Kelvins[1]

[1]To obtain Celsius (C) temperature readings from Fahrenheit (F) readings, use formula: $C = (5/9)(F - 32)$.

To obtain Kelvin (K) readings, use formula: $K = (5/9)(F - 32) + 273.15$.

SYMBOLS AND DEFINITIONS

E	total variance in wind sea
$E_m(f)$	upper bound on energy density in a frequency, f
f	frequency
f_c	low-frequency cutoff
f_p	peak frequency of the spectrum
g	gravitational acceleration
H	depth-controlled wave height (spectral)
H'	linearly shoaled wave height
H_b	breaker height
H_d	depth-limited wave height (monochromatic)
H_s	significant wave height
h	depth
$K_R K_s$	refraction-shoaling coefficient
u	windspeed
α	Phillips' equilibrium coefficient
π	pi
Φ	dimensionless function describing deviation from deepwater equilibrium range
ω_h	dimensionless combination of g, f, and h

A METHOD FOR ESTIMATING DEPTH-LIMITED WAVE ENERGY

by
C. Linwood Vincent

I. INTRODUCTION

This report presents a method for calculating a limit on the total energy of a storm sea in finite-depth water based on characteristics of irregular waves. The total variance, E, in the wave field is parameterized by a wave height parameter, H, defined as

$$H = 4.0(E)^{1/2} \quad (1)$$

The energy in the wave field is directly related to E. H should be recognized as the estimate, based on a Rayleigh distribution, of the significant wave height in deep water. H, defined in this way, is often given as an approximation to the significant wave height (average of highest one-third waves) in shallow water as well, although there is some evidence that the significant wave height may be slightly larger than this estimate. The method presented for estimating H is believed valid for wave conditions where there is spread in the spectrum similar to what might be expected under storm conditions. Estimates for nearly monochromatic swell should follow monochromatic wave theory as discussed in Section 7.11 of the Shore Protection Manual (SPM) (U.S. Army, Corps of Engineers, Coastal Engineering Research Center, 1977)[1]. This report represents a major departure from the SPM methods which are based on regular (monochromatic) waves. There is no clear parallel to this method in the SPM.

It is important to differentiate H from other commonly defined wave parameters in shallow water. H, which will be called the *depth-controlled wave height (spectral)*, simply parameterizes the total wave energy in a spectrum. The significant wave height, H_s, is traditionally defined as the average of the one-third highest waves and is approximately equal to H in deep water. H_b is the breaker height and H_d is the largest individual wave that can exist at a given depth. It should normally be expected that, in an irregular sea, H and H_s will be less than H_b and H_d. H_d will be called the *depth-limited wave height (monochromatic)* to differentiate it from H. H_d is expected to be an estimate of the largest single wave that can occur in a spectrum in water of depth d. Engineering designs that require the largest single wave that can occur should use the SPM methods.

The technical background and an evaluation of this method will be provided in a forthcoming CERC report. A comparison of this method to field data is described in Appendix A.

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[1]U.S. ARMY, CORPS OF ENGINEERS, COASTAL ENGINEERING RESEARCH CENTER, *Shore Protection Manual*, 3d ed., Vols. I, II, and III, Stock No. 008-022-00113-1, U.S. Government Printing Office, Washington, D.C., 1977, 1,262 pp.

II. BACKGROUND

Research by Kitaigorodskii, Krasitskii, and Zaslavskii (1975)[2] has provided an equation for the maximum energy density in a frequency component of a wave spectrum in finite depth

$$E_m(f) = \frac{\alpha g^2 \, 2f^{-5} \, \Phi(\omega_h)}{(2\pi)^4} \tag{2}$$

where

π = 3.1415

f = frequency

g = gravitational acceleration

h = depth

ω_h = dimensionless parameter defined as

$$\omega_h = 2\pi f \left(\frac{h}{g}\right)^{1/2} \tag{3}$$

Φ = dimensionless function of ω_h which varies monotonically from 0 to 1

α = a function of the wave field

Equation (2) has been shown in a number of studies to be an excellent estimate of the upper bound on energy density as a function of f.

E_m represents an upper bound for energy density as a function of frequency. An estimate of an upper bound on the total variance or energy in the wave field can be obtained by integrating equation (2) over the frequencies containing wave energy. This estimate of total energy can be used in equation (1) to obtain an estimate of H. If f_c denotes the lower frequency bounding the energy containing frequencies, then

$$H = 4\left[\int_{f_c}^{\infty} E_m(f) \, df\right]^{1/2} \tag{4}$$

For practical purposes the high-frequency bound for integration in equation (4) is taken as 1 hertz rather than infinity, because for most cases there is little energy beyond 1 hertz in comparison to that below 1 hertz. Clearly, H will vary with depth, h, the lower cutoff frequency, f_c, and the spectral parameter, α; therefore, the notation $H(f_c, h, \alpha)$ will be used. Table 1 presents values of $H(f_c, h, \alpha)$ in feet for values of f_c from 0.05 to 0.34 hertz in steps of 0.01 hertz and for depths of 3 to 60 feet (1 to 20 meters) in 3-foot (1 meter) intervals for a fixed value of $\alpha = 0.0081$. (A metric

[2]KITAIGORODSKII, S.A., KRASITSKII, V.P., and ZASLAVSKII, M.M., "Phillips Theory of the Equilibrium Range in the Spectra of Wind-generated Gravity Waves," *Journal of Physical Oceanography*, Vol. 5, 1975, pp. 410-420.

Table 1. Depth-controlled height, H (f_c, h, 0.0081), as a function of lower frequency cutoff, f_c, period, T, and depth, h.

f_c	T	Depth-controlled height, H (ft)																			
		Depth, h (ft)																			
(hz)	(s)	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
0.05	20.0	5.53	7.93	9.70	11.19	12.49	13.66	14.74	15.72	16.65	17.53	18.36	19.14	19.89	20.61	21.30	21.96	22.61	23.22	23.83	24.40
0.06	16.7	4.68	6.60	8.07	9.30	10.37	11.33	12.22	13.03	13.79	14.51	15.18	15.82	16.43	17.01	17.57	18.10	18.62	19.11	19.59	20.05
0.07	14.3	4.01	5.65	6.90	7.94	8.85	9.67	10.42	11 10	11.74	12.33	12.89	13.43	13.93	14.41	14.87	15.31	15.73	16.14	16.52	16.90
0.08	12.5	3.50	4.93	6.02	6.92	7.71	8.41	9.05	9.64	10.18	10.69	11.16	11.61	12.04	12.44	12.82	13.19	13.54	13.87	14.19	14.50
0.09	11.1	3.11	4.38	5.33	6.13	6.82	7.43	7.98	8.49	8.96	9.40	9.80	10.18	10.54	10.89	11.21	11.51	11.80	12.09	12.34	12.59
0.10	10.0	2.79	3.93	4.78	5.49	6.10	6.64	7.12	7.57	7.98	8.35	8.70	9.03	9.34	9.62	9.90	10.15	10.39	10.62	10.84	11.04
0.11	9.1	2.54	3.56	4.33	4.96	5.51	5.99	6.42	6.81	7.16	7.49	7.79	8.07	8.33	8.58	8.80	9.02	9.22	9.40	9.58	9.75
0.12	8.3	2.32	3.25	3.95	4.52	5.01	5.44	15.82	6.16	6.47	6.76	7.02	7.26	7.48	7.69	7.88	8.06	8.22	8.37	8.51	8.65
0.13	7.7	2.14	2.99	3.63	4.15	4.59	4.97	5.31	5.61	5.89	6.13	6.36	6.56	6.75	6.93	7.08	7.23	7.36	7.48	7.60	7.70
0.14	7.1	1.98	2.77	3.35	3.82	4.22	4.57	4.87	5.14	5.38	5.59	5.78	5.96	6.12	6.26	6.39	6.51	6.61	6.71	6.79	6.87
0.15	6.7	1.84	2.58	3.11	3.54	3.90	4.21	4.48	4.72	4.93	5.11	5.28	5.43	5.56	5.67	5.78	5.87	5.95	6.03	6.09	6.15
0.16	6.3	1.73	2.40	2.90	3.29	3.62	3.90	4.14	4.35	4.53	4.69	4.83	4.95	5.06	5.15	5.24	5.31	5.37	5.43	5.47	5.51
0.17	5.9	1.62	2.25	2.71	3.07	3.37	3.62	3.83	4.02	4.17	4.31	4.43	4.53	4.62	4.69	4.75	4.81	4.86	4.89	4.93	4.96
0.18	5.6	1.53	2.12	2.54	2.87	3.14	3.37	3.56	3.72	3.85	3.97	4.07	4.15	4.22	4.27	4.32	4.36	4.40	4.43	4.45	4.46
0.19	5.3	1.44	2.00	2.39	2.70	2.94	3.14	3.31	3.45	3.56	3.66	3.74	3.80	3.86	3.90	3.94	3.97	3.99	4.01	4.02	4.04
0.20	5.0	1.36	1.89	2.25	2.53	2.76	2.94	3.08	3.20	3.30	3.38	3.44	3.49	3.53	3.57	3.59	3.61	3.63	3.64	3.65	3.66
0.21	4.8	1.30	1.79	2.13	2.39	2.59	2.75	2.88	2.98	3.06	3.12	3.17	3.21	3.24	3.27	3.28	3.30	3.31	3.32	3.33	3.33
0.22	4.5	1.23	1.70	2.01	2.25	2.43	2.57	2.69	2.77	2.84	2.89	2.93	2.96	2.98	3.00	3.01	3.02	3.03	3.03	3.03	3.04
0.23	4.3	1.17	1.61	1.91	2.12	2.29	2.41	2.51	2.58	2.64	2.68	2.71	2.73	2.74	2.76	2.77	2.77	2.77	2.78	2.78	2.78
0.24	4.2	1.12	1.53	1.81	2.01	2.16	2.27	2.35	2.41	2.45	2.48	2.50	2.52	2.53	2.54	2.55	2.55	2.55	2.55	2.55	2.55
0.25	4.0	1.07	1.46	1.72	1.90	2.03	2.13	2.20	2.25	2.28	2.30	2.32	2.33	2.34	2.35	2.35	2.35	2.35	2.35	2.35	2.35
0.26	3.8	1.03	1.40	1.64	1.80	1.92	2.00	2.06	2.10	2.13	2.14	2.16	2.16	2.17	2.17	2.17	2.17	2.17	2.17	2.17	2.17
0.27	3.7	0.98	1.33	1.56	1.71	1.81	1.88	1.93	1.96	1.98	2.00	2.01	2.01	2.01	2.01	2.01	2.01	2.01	2.01	2.01	2.01
0.28	3.6	0.95	1.28	1.48	1.62	1.71	1.77	1.81	1.84	1.85	1.86	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87
0.29	3.4	0.91	1.22	1.41	1.54	1.62	1.67	1.70	1.72	1.73	1.74	1.74	1.74	1.74	1.74	1.74	1.74	1.74	1.74	1.74	1.74
0.30	3.3	0.87	1.17	1.35	1.46	1.53	1.57	1.60	1.61	1.62	1.63	1.63	1.63	1.63	1.63	1.63	1.63	1.63	1.63	1.63	1.63
0.31	3.2	0.84	1.12	1.29	1.39	1.45	1.48	1.50	1.52	1.52	1.52	1.52	1.52	1.53	1.53	1.53	1.53	1.53	1.53	1.53	1.53
0.32	3.1	0.81	1.08	1.23	1.32	1.37	1.40	1.42	1.43	1.43	1.43	1.43	1.43	1.43	1.43	1.43	1.43	1.43	1.43	1.43	1.43
0.33	3.0	0.78	1.03	1.18	1.25	1.30	1.32	1.34	1.34	1.34	1.34	1.34	1.34	1.34	1.34	1.34	1.34	1.34	1.34	1.34	1.34
0.34	2.9	0.75	0.99	1.12	1.19	1.23	1.25	1.26	1.26	1.26	1.26	1.27	1.27	1.27	1.27	1.27	1.27	1.27	1.27	1.27	1.27

version of the table is in Appendix B.) The parameter α is related to dimensionless fetch and can be either obtained from a measured deepwater spectrum or can be inferred as a function of a peak frequency of a spectrum and windspeed. The adjustment of $H(f_c, h, 0.0081)$ for variation in α is discussed in the next section.

If the parameter, ω_h, defined in equation (3) is one or less for the main energy containing frequencies in a sea state, the $H(f_c, h, 0.0081)$ can be approximated by

$$H(f_c, h, 0.0081) = \frac{0.161\ h^{1/2}}{f_c} \text{ for } h \text{ in feet}$$
$$= \frac{0.089\ h^{1/2}}{f_c} \text{ for } h \text{ in meters} \qquad (5)$$

This equation predicts that in areas where the wave energy is controlled by depth, H varies with the square root of depth, not approximately *linearly* with depth as does the monochromatic H_d.

III. SIMPLIFIED METHOD

To use Table 1 or equation (5), estimates of f_c and α are necessary to estimate H in depth, h.

1. Selection of f_c.

f_c is defined as the lowest frequency in which there is appreciable wave energy. Typical storm spectra in shallow water (60 feet or less) have very sharp peaks. If there is a given spectrum offshore where the values of H are desired in a depth, h, it is recommended that f_c be set to 90 percent of f_p, the peak frequency of the sea spectrum.

$$f_c = 0.9\ f_p \qquad (6)$$

If more conservatism is required, f_c can be set lower. If no spectral data are available at a site, then f_c can be estimated either by estimating f via hindcast curves or by selecting a reasonable but conservative value of f_c based on similar conditions elsewhere.

2. Selection of α.

The parameter of α can be directly estimated from measured spectrum by fitting equation (2) to the spectrum. If measurements are unavailable, α can be estimated by fitting equation (2) to good quality hindcast spectra. If neither hindcasts nor measured data are available, then α can be estimated from Hasselmann, et al. (1973)[3]

$$\alpha = 0.076 \left(\frac{gF}{u^2} \right)^{-0.22}$$

[3]HASSELMANN, K., et al., "Measurements of Wind-Wave Growth and Swell Decay During the Joint North Sea Wave Project JONSWAP," Deutsches Hydrographisches Institut, Hamburg Germany, 1973.

where f is fetch and u is windspeed. For $\alpha \neq 0.0081$

$$H(f_c, h, \alpha) = H(fc, h, 0.0081) \left(\frac{\alpha}{0.0081}\right)^{1/2} \quad (7)$$

An alternate way to estimate $(\alpha/0.0081)^{1/2}$ is to use Table 2 where $(\alpha/0.0081)^{1/2}$ is provided for peak frequencies from 0.05 to 0.34 and windspeeds from 10 to 100 miles per hour (5 to 50 meters per second). (See App. B for metric version of tables.) Appendix A describes the comparison of this method to field data.

IV. EXAMPLE PROBLEMS

* * * * * * * * * * * * * * * * EXAMPLE PROBLEM 1 * * * * * * * * * * * * * * * *

GIVEN: A wave spectrum measured offshore has a significant height of 18 feet (6 meters) with a peak frequency $f_p = 0.08$ and α value of 0.0101.

FIND: The depth-controlled wave height (spectral) H in 45, 30, 15, and 3 feet (15, 10, 5, and 1 meter) of water.

SOLUTION: Calculate $f_c = 0.9\ f_p = 0.072$, using 0.07. From Table 1 using, $\alpha = 0.0081$, find

H = 14.9 feet (4.5 meters) in 45 feet

H = 12.3 feet (3.8 meters) in 30 feet

H = 8.9 feet (2.7 meters) in 15 feet

H = 4.0 feet (1.2 meters) in 3 feet

These values must be adjusted to $(\alpha/0.0081)^{1/2} = 1.11$, but the correction is very small in each case. Examination of field data indicates that in depths less than 13 feet (4 meters), wave spectral densities in frequencies less than 0.1 hertz can be substantially smaller than the upper bound value in equation (2), probably due to frictional effects and turbulence; $H(f_c, h, \alpha)$ can be an overestimate. Also note that h representing actual water depth including tide, surge, and wave setup is used.

* * * * * * * * * * * * * * * * EXAMPLE PROBLEM 2 * * * * * * * * * * * * * * * *

GIVEN: Hindcasts on a lake indicate that under design storm conditions, peak frequencies were not expected to be any lower than 0.17 hertz for windspeeds of 68 miles per hour (30 meters per second).

FIND: The depth-controlled (spectral) wave heights in 30, 15, 10, and 3 feet (10, 5, 3, and 1 meter) of water.

SOLUTION: Calculate $f_c = 0.9\ f_p = 0.9(0.17) = 0.153$, using 0.15. From Table 1 using $\alpha = 0.0081$, find

H = 5.1 feet (1.6 meters) in 30 feet

H = 3.9 feet (1.2 meters) in 15 feet

H = 3.2 feet (1.0 meter) in 10 feet

H = 1.8 feet (0.6 meter) in 3 feet

Table 2. Values of $(\alpha/0.0081)^{1/2}$ as function of spectral peak frequency, f_p, period, T, windspeed, u.

f_p	T	Windspeed, u (mi/hr)									
(hz)	(s)	10	20	30	40	50	60	70	80	90	100
0.05	20.0	0.582	0.731	0.836	0.919	0.989	1.050	1.105	1.155	1.201	1.243
0.06	16.7	0.618	0.776	0.887	0.976	1.050	1.116	1.174	1.227	1.275	1.320
0.07	14.3	0.650	0.817	0.934	1.027	1.105	1.174	1.235	1.291	1.342	1.389
0.08	12.5	0.679	0.854	0.976	1.073	1.155	1.227	1.291	1.349	1.402	1.452
0.09	11.1	0.706	0.887	1.015	1.116	1.201	1.275	1.342	1.402	1.458	1.509
0.10	10.0	0.731	0.919	1.050	1.155	1.243	1.320	1.389	1.452	1.509	1.563
0.11	9.1	0.754	0.948	1.084	1.192	1.283	1.363	1.434	1.489	1.558	1.613
0.12	8.3	0.776	0.976	1.116	1.227	1.320	1.402	1.475	1.542	1.603	1.660
0.13	7.7	0.797	1.002	1.145	1.260	1.356	1.440	1.515	1.583	1.646	1.704
0.14	7.1	0.817	1.027	1.174	1.291	1.389	1.475	1.552	1.622	1.687	1.746
0.15	6.7	0.836	1.050	1.201	1.320	1.421	1.509	1.588	1.660	1.726	1.787
0.16	6.3	0.854	1.073	1.227	1.349	1.452	1.542	1.622	1.696	1.763	1.825
0.17	5.9	0.871	1.095	1.251	1.376	1.481	1.573	1.655	1.730	1.798	1.862
0.18	5.6	0.887	1.116	1.275	1.402	1.509	1.603	1.687	1.763	1.833	1.897
0.19	5.3	0.903	1.136	1.298	1.438	1.537	1.632	1.717	1.794	1.866	1.932
0.20	5.0	0.919	1.155	1.320	1.452	1.563	1.660	1.746	1.825	1.897	1.965
0.21	4.8	0.934	1.174	1.342	1.475	1.588	1.687	1.775	1.855	1.928	1.996
0.22	4.5	0.948	1.192	1.363	1.498	1.613	1.713	1.802	1.883	1.958	2.027
0.23	4.3	0.962	1.210	1.383	1.520	1.637	1.738	1.829	1.911	1.987	2.057
0.24	4.2	0.976	1.227	1.402	1.542	1.660	1.763	1.855	1.938	2.015	2.086
0.25	4.0	0.989	1.243	1.421	1.563	1.682	1.787	1.880	1.965	2.042	2.115
0.26	3.8	1.002	1.260	1.440	1.583	1.704	1.810	1.904	1.990	2.069	2.142
0.27	3.7	1.015	1.275	1.458	1.603	1.726	1.833	1.928	2.015	2.095	2.169
0.28	3.6	1.027	1.291	1.475	1.622	1.746	1.855	1.951	2.039	2.120	2.195
0.29	3.4	1.039	1.306	1.493	1.641	1.767	1.876	1.974	2.063	2.145	2.221
0.30	3.3	1.050	1.320	1.509	1.660	1.787	1.897	1.996	2.086	2.169	2.246
0.31	3.2	1.062	1.335	1.526	1.678	1.806	1.918	2.018	2.109	2.193	2.270
0.32	3.1	1.073	1.349	1.542	1.696	1.825	1.938	2.039	2.131	2.216	2.294
0.33	3.0	1.084	1.363	1.558	1.713	1.844	1.958	2.060	2.153	2.238	2.318
0.34	2.9	1.095	1.376	1.573	1.730	1.862	1.977	2.081	2.174	2.261	2.340

To adjust for the proper value of α use Table 2 which indicates that $(\alpha/0.0081)^{1/2}$ for f = 0.17 hertz and u = 68 miles per hour is 1.63. The above wave heights must be multiplied by 1.63 yielding 8.3, 6.3, 5.2, and 2.9 feet (2.6, 2.0, 1.6, and 0.9 meter).

V. DISCUSSION

The two example problems indicate that the depth-controlled wave height (spectral) depends significantly on the values chosen for α and f_c. On the lake, the waves have a higher α value but because of fetch limitations, f_p cannot be very low. As a result, there is less spread in energy over the spectrum and the limiting form (eq. 2) is only integrated over frequencies where the energy density is relatively small compared to cases where f_p is lower and E_m is much larger. The method presented here indicates that the upper bound on wave energy and an estimate of the significant height, H_s, by H varies with depth (nonlinearly), the peak frequency associated with the waves, and a parameter α associated with the wave generation process. The method allows an estimation of the depth-limited conditions of $H = 4(E)^{1/2}$ to be based on an analysis of the wave generation conditions.

The difference between H (directly related to the energy) and H_d (based on the largest single wave that can occur) is significant for most wave conditions and especially when the site has short fetches. It is recommended that H_d only be used where it is necessary to determine the largest individual wave or if the wave conditions are nearly monochromatic. In all other cases, and in particular, where an estimate of the energy of the wave field is required, the method in this report may be expected to provide a more accurate (though less conservative) estimate.

VI. SUMMARY

A method has been presented for calculating a maximum value for $H = 4(E)^{1/2}$ for wind sea situations where depth is the controlling factor. Estimation requires input of depth, a lower bound frequency, and parameter α typical of storm spectra at the site. Methods for estimating the latter two parameters are also provided. The results indicate that, in the shallow-water limit, H (which is also an approximation to the shallow-water value H_s) is proportional to the square root of depth. The values obtained can be significantly less than the monochromatic depth-limited wave height, H_d, which is taken to be the upper bound of individual single waves in the spectrum. Again it is important to emphasize that the H defined here is directly related to the total energy of the wind sea and approximates the average of the highest one-third waves. The monochromatic value H_d defined in SPM better approximates the highest individual wave that might be expected in that depth. The selection of which approximation to use will depend on the design application.

APPENDIX A

- PREDICTION CAPABILITY OF METHOD

Because the method presented in this CETA is significantly different from the procedures in the SPM, summarized data are presented here to illustrate the method's prediction capability. Irregular wave conditions with a spectral shape similar to wind sea spectra were mechanically generated in a wave tank at CERC. The waves were allowed to propagate up a 1 on 30 slope and break. Wave staffs located along the tank were used to measure the waves. Wave spectra and the wave height $H = 4\sigma = 4(E)^{1/2}$ were calculated. Figure A-1 provides an example showing measured H (H obtained using this CETA) and the value of H projected by linear shoaling theory, $H' = K_S H_o$. The SPM methods would predict the value H'. The method in this CETA appears to be a better estimate of the quantity H.

Figure A-2 provides data from a storm of 25 October 1980 at the Field Research Facility (FRF) in Duck, North Carolina. The measured values of $H = 4\sigma$ are plotted against the square root of water depth, $(h)^{1/2}$. Also plotted are (1) the upper limit for H for the maximum α and f_c condition observed, and (2) a monochromatic determined breaker height which considers the shore steepness, water depth, and wave period. In most cases H_b is greater than H. A suggested approximation of $H = 0.5h$ is also plotted. This relationship is adequate at the deeper end of the pier but is an underestimate at the shallower depths.

The bathymetry under the pier is somewhat distorted (Fig. A-3,a). A refraction and shoaling analysis of a 12-second wave approaching from the primary direction of waves of the October storm was performed and the joint refraction-shoaling coefficient, $K_R K_S$, is shown in Figure A-3(b). Refraction and shoaling create regions of higher waves to either side of the pier. However, shoaling predominates and even under the pier, $K_R K_S$ is greater than 1.1. During the part of the October storm plotted in Figure A-2, waves offshore were in excess of 4.2 meters, and waves along the pier would be expected to reach the monochromatic breaking limit unless some other process is acting.

Figure A-4 provides plots of spectra at 36-, 8-, 7-, 4- and 2-meter depths during a storm at the FRF in December 1980. The expected upper bound on spectral density for the 7-, 4-, and 2-meter depths are plotted also. The figure indicates the degree and location of energy loss in the spectrum and the degree of approximation of the theory used in development of this CETA.

The method in this CETA represents a simple approximation of estimating depth-controlled wave energy. The method does not consider any of a number of mechanisms important to predicting precisely the shape of the wave spectrum but still provides what appears to be a useful approximation to the quantity $H = 4\sigma = 4(E)^{1/2}$.

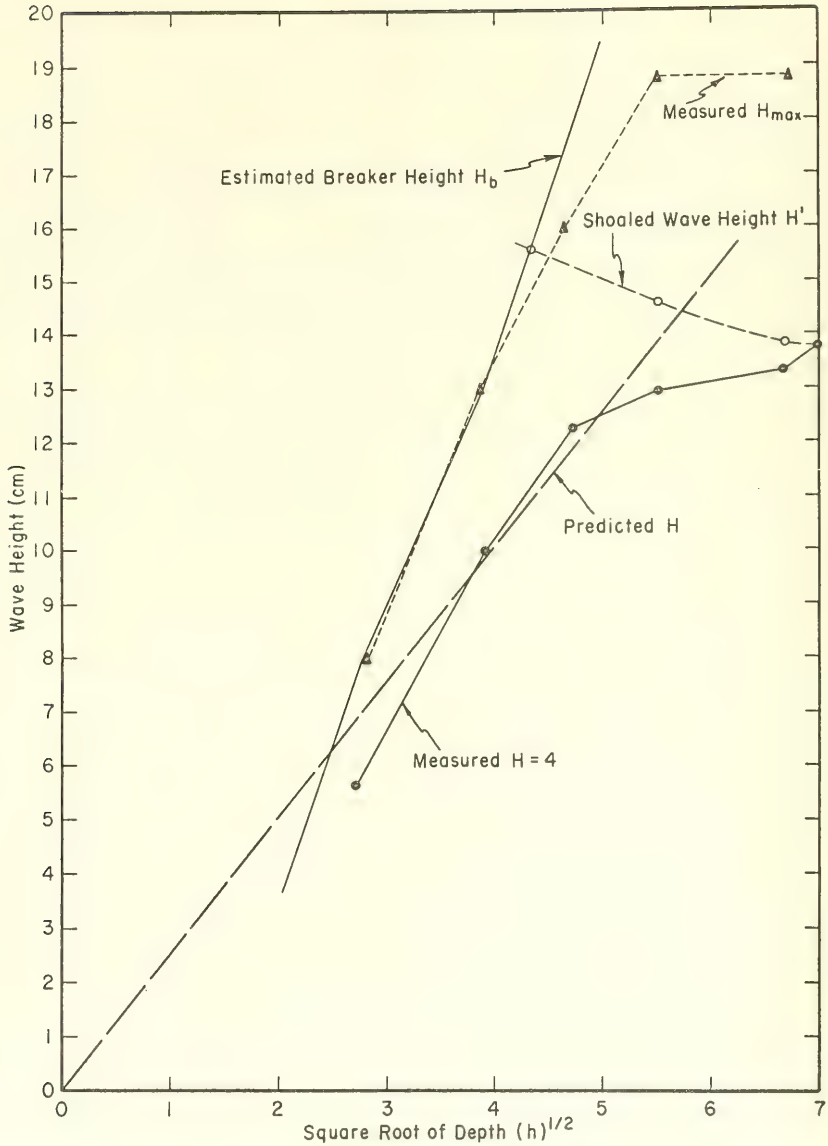

Figure A-1. Laboratory data. The wave height parameter $H = 4\sigma$ measured in a CERC wave tank along a 1:30 slope is compared to predicted value H as a function of $(h)^{1/2}$. Also shown is the linearly shoaled wave height H' and the breaker height H_b. The maximum individual measured wave height, H_{max}, is shown and appears to follow H_b.

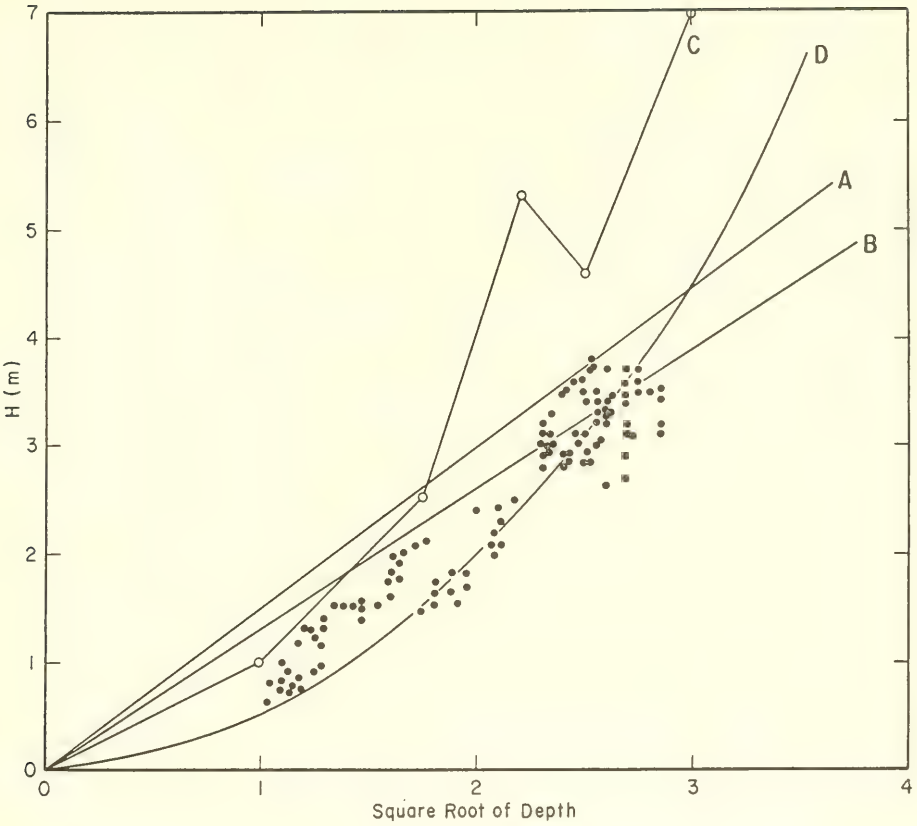

Figure A-2. Field data, 25 October 1980. $H = 4\sigma$, measured at the FRF, is plotted against the square root of water depth, $(h)^{1/2}$. Also plotted are the values of H estimated by the method of this CETA given α at 36 meters (line A) and α at 9 meters (line B). The breaker height, H_b, is plotted (line C) with bottom slope variations accounted for. An estimate of $H = 0.5h$ is also given (line D). During the storm, maximum wave heights of 5.6 meters were observed in 7- to 9-meter depths.

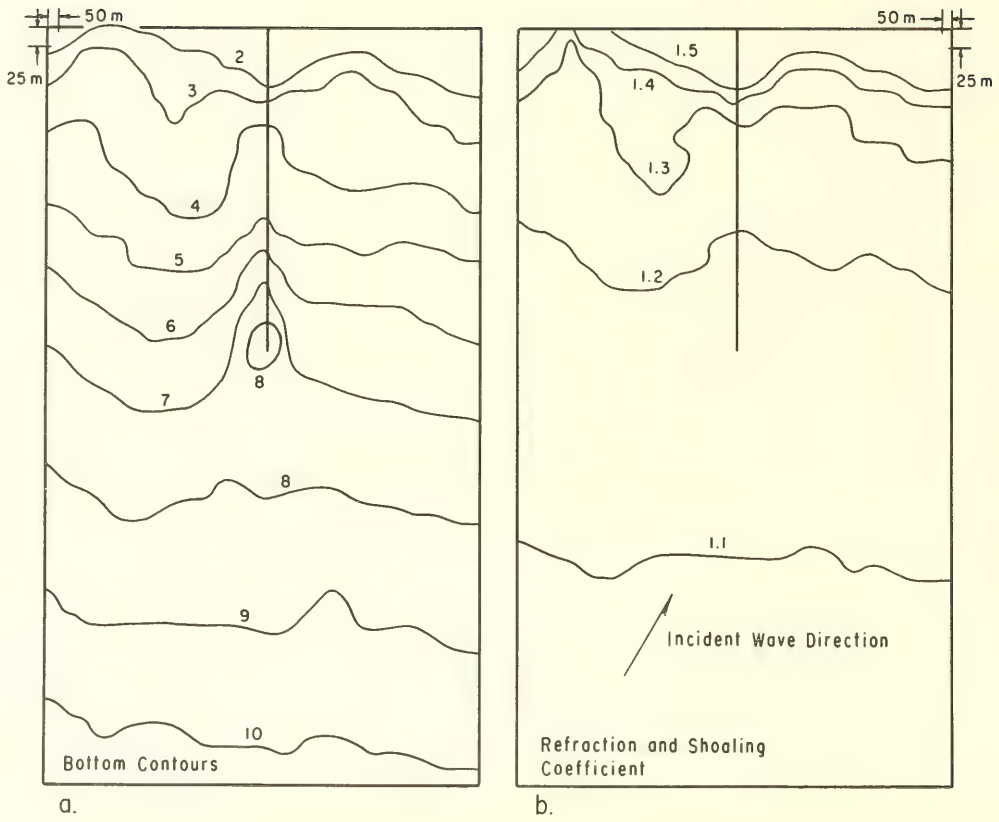

Figure A-3. Refraction and shoaling patterns at the FRF. In (a) bottom contours around the FRF are shown. In (b) the refraction-shoaling coefficient is shown for 12-second waves from due east. The X-Y scale of the plot is distorted.

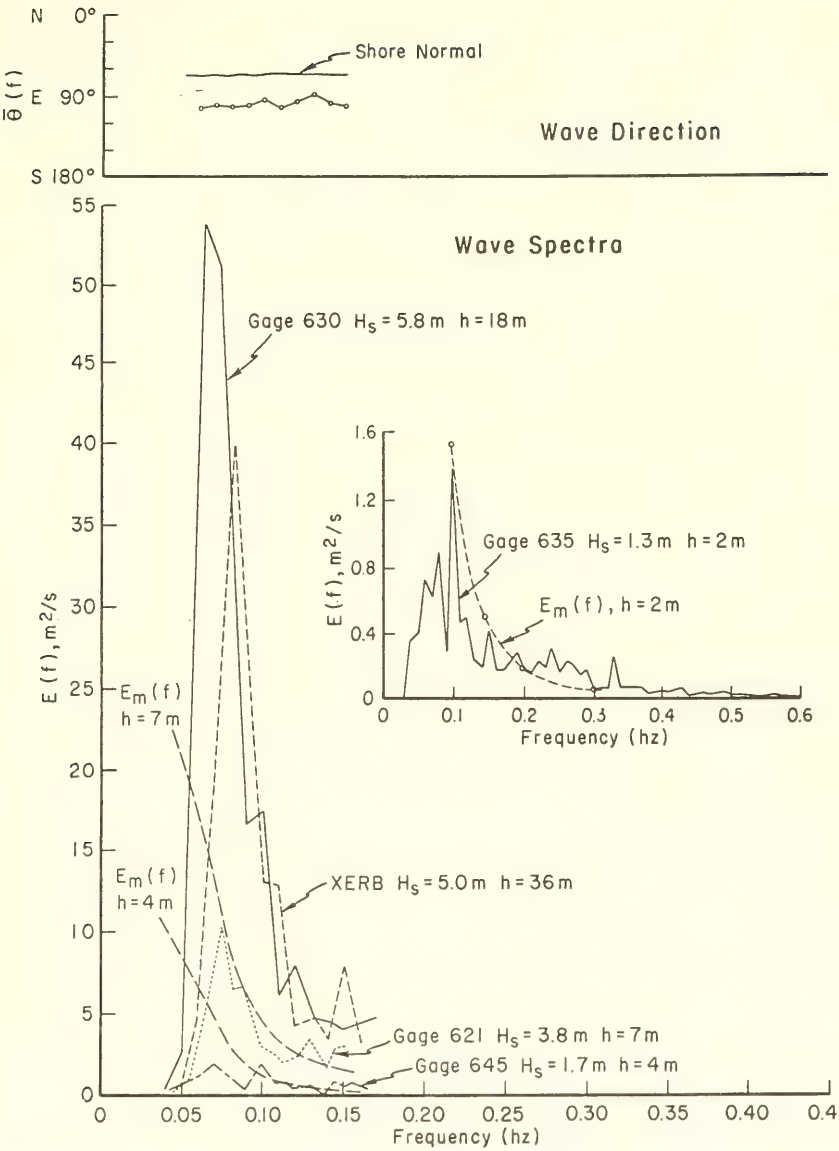
N 0°
E 90°
S 180°
θ̄(f)
Shore Normal
Wave Direction
Wave Spectra
Gage 630 Hs = 5.8 m h = 18 m
Gage 635 Hs = 1.3 m h = 2 m
Em(f), h = 2 m
E(f), m²/s
Frequency (hz)
Em(f) h = 7 m
Em(f) h = 4 m
XERB Hs = 5.0 m h = 36 m
Gage 621 Hs = 3.8 m h = 7 m
Gage 645 Hs = 1.7 m h = 4 m

Figure A-4. Wave spectra during storm of 28 December 1980. Wave spectra from 36-, 7-, 4-, and 2-meter depths are plotted with the predicted value of $E_m(f)$ for depths of 7 meters or less. The value of E_m appears to be an estimate for parts of the spectrum with frequencies less than $2f_m$, where f_m is the peak of the spectrum. Above $2f_m$ the value often tends to be an underestimate because the spectral values are related to harmonics of the dominant wave. However, the differences tend to be small. The wave direction $\bar{\theta}$ at 36-meter depth is plotted by frequency.

APPENDIX B

METRIC VERSION OF TEXT TABLES

Table B-1. Depth controlled height, H (f_c, h, 0.0081), as function of lower frequency cutoff, f_c, period, T, and depth, h.

f_c (hz)	T (s)	Depth (m)																			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.05	20.0	1.79	2.53	3.09	3.56	3.98	4.35	4.69	5.01	5.30	5.58	5.84	6.09	6.33	6.56	6.77	6.98	7.18	7.38	7.57	7.75
0.06	16.7	1.49	2.10	2.57	2.96	3.30	3.61	3.89	4.15	4.39	4.62	4.83	5.03	5.22	5.41	5.58	5.75	5.91	6.07	6.22	6.36
0.07	14.3	1.28	1.80	2.20	2.53	2.82	3.08	3.31	3.53	3.73	3.92	4.10	4.26	4.42	4.57	4.72	4.86	4.99	5.11	5.23	5.35
0.08	12.5	1.12	1.57	1.92	2.20	2.45	2.68	2.88	3.06	3.23	3.39	3.54	3.68	3.82	3.94	4.06	4.18	4.29	4.39	4.49	4.58
0.09	11.1	0.99	1.39	1.70	1.95	2.17	2.36	2.54	2.70	2.84	2.98	3.11	3.23	3.34	3.44	3.54	3.64	3.73	3.81	3.90	3.97
0.10	10.0	0.89	1.25	1.52	1.75	1.94	2.11	2.26	2.40	2.53	2.65	2.76	2.86	2.95	3.04	3.12	3.20	3.28	3.35	3.41	3.47
0.11	9.1	0.81	1.13	1.38	1.58	1.75	1.90	2.03	2.16	2.27	2.37	2.46	2.55	2.63	2.70	2.77	2.84	2.90	2.95	3.01	3.06
0.12	8.3	0.74	1.04	1.26	1.44	1.59	1.72	1.84	1.95	2.05	2.13	2.22	2.29	2.36	2.42	2.48	2.53	2.58	2.62	2.66	2.70
0.13	7.7	0.68	0.95	1.15	1.32	1.45	1.57	1.68	1.77	1.86	1.93	2.00	2.06	2.12	2.17	2.22	2.26	2.30	2.34	2.37	2.40
0.14	7.1	0.63	0.88	1.07	1.21	1.34	1.44	1.54	1.62	1.69	1.76	1.82	1.87	1.92	1.96	2.00	2.03	2.06	2.09	2.11	2.13
0.15	6.7	0.59	0.82	0.99	1.12	1.24	1.33	1.41	1.49	1.55	1.61	1.66	1.70	1.74	1.77	1.80	1.83	1.85	1.87	1.89	1.90
0.16	6.3	0.55	0.76	0.92	1.04	1.14	1.23	1.30	1.37	1.42	1.47	1.51	1.55	1.58	1.60	1.63	1.65	1.66	1.68	1.69	1.70
0.17	5.9	0.52	0.72	0.86	0.97	1.06	1.14	1.21	1.26	1.31	1.35	1.38	1.41	1.43	1.45	1.47	1.49	1.50	1.51	1.52	1.52
0.18	5.6	0.49	0.67	0.81	0.91	0.99	1.06	1.12	1.17	1.20	1.24	1.27	1.29	1.31	1.32	1.33	1.34	1.35	1.36	1.36	1.37
0.19	5.3	0.46	0.63	0.76	0.85	0.93	0.99	1.04	1.08	1.11	1.14	1.16	1.18	1.19	1.20	1.21	1.22	1.22	1.23	1.23	1.24
0.20	5.0	0.43	0.60	0.71	0.80	0.87	0.92	0.96	1.00	1.03	1.05	1.07	1.08	1.09	1.10	1.10	1.11	1.11	1.12	1.12	1.12
0.21	4.8	0.41	0.57	0.67	0.75	0.81	0.86	0.90	0.93	0.95	0.97	0.98	0.99	1.00	1.00	1.01	1.01	1.01	1.01	1.01	1.02
0.22	4.5	0.39	0.54	0.64	0.71	0.76	0.80	0.84	0.86	0.88	0.89	0.90	0.91	0.91	0.92	0.92	0.92	0.92	0.92	0.93	0.93
0.23	4.3	0.37	0.51	0.60	0.67	0.72	0.75	0.78	0.80	0.81	0.82	0.83	0.84	0.84	0.84	0.85	0.85	0.85	0.85	0.85	0.85
0.24	4.2	0.36	0.49	0.57	0.63	0.67	0.71	0.73	0.74	0.76	0.76	0.77	0.77	0.78	0.78	0.78	0.78	0.78	0.78	0.78	0.78
0.25	4.0	0.34	0.46	0.54	0.60	0.63	0.66	0.68	0.69	0.70	0.71	0.71	0.71	0.72	0.72	0.72	0.72	0.72	0.72	0.72	0.72
0.26	3.8	0.33	0.44	0.51	0.56	0.60	0.62	0.64	0.65	0.65	0.66	0.66	0.66	0.66	0.66	0.66	0.66	0.66	0.66	0.66	0.66
0.27	3.7	0.31	0.42	0.49	0.53	0.56	0.58	0.60	0.60	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61
0.28	3.6	0.30	0.40	0.47	0.51	0.53	0.55	0.56	0.56	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57
0.29	3.4	0.29	0.39	0.44	0.48	0.50	0.51	0.52	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53
0.30	3.3	0.28	0.37	0.42	0.45	0.47	0.48	0.49	0.49	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
0.31	3.2	0.27	0.35	0.40	0.43	0.45	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46
0.32	3.1	0.26	0.34	0.38	0.41	0.42	0.43	0.43	0.44	0.44	0.44	0.44	0.44	0.44	0.44	0.44	0.44	0.44	0.44	0.44	0.44
0.33	3.0	0.25	0.33	0.37	0.39	0.40	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41
0.34	2.9	0.24	0.31	0.35	0.37	0.38	0.38	0.38	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39

Table B-2. Values of $(\alpha/0.0081)^{1/2}$ as function of spectral peak frequency, f_p, period, T, and windspeed, u.

f_p	T	Windspeed, u (m/s)									
(hz)	(s)	5	10	15	20	25	30	35	40	45	50
0.05	20.0	0.60	0.76	0.87	0.95	1.03	1.09	1.15	1.20	1.25	1.29
0.06	16.7	0.64	0.81	0.92	1.01	1.09	1.16	1.22	1.27	1.32	1.37
0.07	14.3	0.67	0.85	0.97	1.07	1.15	1.22	1.28	1.34	1.39	1.44
0.08	12.5	0.71	0.89	1.01	1.11	1.20	1.27	1.34	1.40	1.46	1.51
0.09	11.1	0.73	0.92	1.05	1.16	1.25	1.32	1.39	1.46	1.51	1.57
0.10	10.0	0.76	0.95	1.09	1.20	1.29	1.37	1.44	1.51	1.57	1.62
0.11	9.1	0.78	0.98	1.13	1.24	1.33	1.41	1.49	1.56	1.62	1.67
0.12	8.3	0.81	1.01	1.16	1.27	1.37	1.46	1.53	1.60	1.66	1.72
0.13	7.7	0.83	1.04	1.19	1.31	1.41	1.49	1.57	1.64	1.71	1.77
0.14	7.1	0.85	1.07	1.22	1.34	1.44	1.53	1.61	1.68	1.75	1.81
0.15	6.7	0.87	1.09	1.25	1.37	1.48	1.57	1.65	1.72	1.79	1.85
0.16	6.3	0.89	1.11	1.27	1.40	1.51	1.60	1.68	1.76	1.83	1.89
0.17	5.9	0.90	1.14	1.30	1.43	1.54	1.63	1.72	1.80	1.87	1.93
0.18	5.6	0.92	1.16	1.32	1.46	1.57	1.66	1.75	1.83	1.90	1.97
0.19	5.3	0.94	1.18	1.35	1.48	1.60	1.69	1.78	1.86	1.94	2.01
0.20	5.0	0.95	1.20	1.37	1.51	1.62	1.72	1.81	1.89	1.97	2.04
0.21	4.8	0.97	1.22	1.39	1.53	1.65	1.75	1.84	1.93	2.00	2.07
0.22	4.5	0.98	1.24	1.41	1.56	1.67	1.78	1.87	1.96	2.03	2.10
0.23	4.3	1.00	1.26	1.44	1.58	1.70	1.80	1.90	1.98	2.06	2.14
0.24	4.2	1.01	1.27	1.46	1.60	1.72	1.83	1.93	2.01	2.09	2.17
0.25	4.0	1.03	1.29	1.48	1.62	1.75	1.85	1.95	2.04	2.12	2.20
0.26	3.8	1.04	1.31	1.49	1.64	1.77	1.88	1.98	2.07	2.15	2.22
0.27	3.7	1.05	1.32	1.51	1.66	1.79	1.90	2.00	2.09	2.17	2.25
0.28	3.6	1.07	1.34	1.53	1.68	1.81	1.93	2.03	2.12	2.20	2.28
0.29	3.4	1.08	1.36	1.55	1.70	1.83	1.95	2.05	2.14	2.23	2.31
0.30	3.3	1.09	1.37	1.57	1.72	1.85	1.97	2.07	2.17	2.25	2.33
0.31	3.2	1.10	1.39	1.58	1.74	1.87	1.99	2.10	2.19	2.28	2.36
0.32	3.1	1.11	1.40	1.60	1.76	1.89	2.01	2.12	2.21	2.30	2.38
0.33	3.0	1.13	1.41	1.62	1.78	1.91	2.03	2.14	2.24	2.32	2.41
0.34	2.9	1.14	1.42	1.63	1.80	1.93	2.05	2.16	2.26	2.35	2.43

Vincent, Charles L. (Charles Linwood)
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